A Method for Finding the Square of any number

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1 Square of any number

Let N_n be any $n-$ digit number, $n \geq 2$.
Then N_n can be written as:

$$N_n = a_{n-1} \times 10^{n-1} + a_{n-2} \times 10^{n-2} + ... + a_1 \times 10 + a_0, \; N_1 = a_0 \neq 0.$$

Now,

$$N_n = a_{n-1} \times 10^{n-1} + N_{n-1}$$

Squaring on both sides, we have

$$\begin{aligned} N_n^2 &= a_{n-1}^2 \times (10^{n-1})^2 + a_{n-1} \times 10^{n-1} \times N_{n-1} + a_{n-1} \times 10^{n-1} \times N_{n-1} + N_{n-1}^2 \\ &= a_{n-1} \times 10^{n-1}(a_{n-1} \times 10^{n-1} + N_{n-1} + N_{n-1}) + N_{n-1}^2 \\ &= a_{n-1} \times 10^{n-1}(N_n + N_{n-1}) + N_{n-1}^2 \end{aligned}$$

We have the recurrence formula

$$N_n^2 = a_{n-1} \times 10^{n-1}(N_n + N_{n-1}) + N_{n-1}^2 \tag{1}$$

which reduces to

$$N_n^2 = \sum_{j=1}^{n} a_{n-j} \times 10^{n-j}(N_{n-j+1} + N_{n-j}) + N_{n-j}^2, \; n \geq 2, \; N_1 = a_0. \tag{2}$$

Example 1.1 (2-digit number)**.**

$$\begin{aligned} 37^2 &= 3 \times 10(37 + 7) + 7^2 \\ &= 1369 \end{aligned}$$

Example 1.2 (3-digit number)**.**

$$\begin{aligned} 736^2 &= 7 \times 10^2(736 + 36) + 3 \times 10(36 + 6) + 6^2 \\ &= 541696 \end{aligned}$$

Example 1.3 (4-digit number)**.**

$$\begin{aligned} 8758^2 &= 8 \times 10^3(8758 + 758) + 7 \times 10^2(758 + 58) + 5 \times 10(58 + 8) + 8^2 \\ &= 76702564 \end{aligned}$$